MLE estimation

1) mle estimation for rare events in a fixed number of bernoulli trials

model: $y_1, \ldots, y_n$ iid p_β
where $p = \mathbb{P}(Y = 1) = 1 - \mathbb{P}(Y = 0)$ find:

1. the likelihood function $L(p)$
2. the asymptotic variance of the mle $\widehat{p}$
3. an asymptotic 95% conf int for p
4. a graph of the likelihood function for $p = 2, 1, 0$
5. an asymptotic 95% conf for p if $n = 16$ and $\sum y = 2, 1, 0$ what do you note?
6. generalize this result

2) Reparametrized Model:

model: $y_1, \ldots, y_n$ iid p_β
where $p_\beta = \mathbb{P}(Y = 1) = 1 - \mathbb{P}(Y = 0) = \frac{\exp \beta}{1+\exp \beta}$ find:

1. the log likelihood function for β
2. $\widehat{\beta}$ mle of β.
3. the Fisher information
4. the asymptotic distribution of $\widehat{\beta}$
5. estimate the asymptotic variance of $\widehat{\beta}$
6. estimate an asymptotic 95% confidence interval for β
7. from 6. derive an asymptotic 95% confidence interval for p

3) generalization:

model: $y_1, \ldots, y_n, y_{n+1}, \ldots, y_{n+m}$ iid p
where

$$p_\beta = \mathbb{P}(Y_i = 1) = 1 - \mathbb{P}(Y_i = 0) = \begin{cases} \frac{\exp\beta}{1+\exp\beta} & \text{if } i = 1, \ldots, n \\ \frac{\exp(\beta+\delta)}{1+\exp(\beta+\delta)} & \text{if } i = n+1, \ldots, n+m \end{cases}$$

find:

1. the log likelihood function for (β, δ)
2. $\widehat{\beta}$ mle of β, $\widehat{\delta}$ mle of δ.
3. the Fisher information
4. the asymptotic distribution of $\left(\widehat{\beta}, \widehat{\delta}\right)$
5. estimate the asymptotic covariance matrix of $\left(\widehat{\beta}, \widehat{\delta}\right)$
6. estimate an asymptotic 90% confidence interval for β
7. test for $\mathrm{H}_0 : \delta = 0$