Economet-II SS12

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Final Exam (26.07.12)

1. using a uniform random number generator $U_1, \ldots, U_n$ iid $\sim \mathcal{U}(0,1)$ explain (algorithm) how to:

   a) generate random numbers from

   $$X_1, \ldots, X_n \text{ iid} \sim EXP(\lambda)$$

   with $\lambda = 2$

   b) compute by MC simulation

   $$\int_0^\infty \sqrt{x} e^{-2x} dx$$

   c) give a plausible reason why the method described in **b)** should work.

   d) compute by MC simulation

   $$\int \int_{x+y<1} e^{-\frac{1}{2}\left(x^2+y^2\right)} dx\ dy$$

2. consider the model

   $$x_i = \theta_i + \epsilon_i$$
   $$\theta_i \text{ iid } \sim N\left(\mu, \sigma^2\right),\ \epsilon_i \text{ iid } \sim N\left(0, \sigma_x^2\right)$$
   $$\theta_i \text{ indep } \epsilon_j,\ i, j = 1, \ldots n$$

   a) sketch a DAG (directed acyclic graph) for Bayesian inference on $\mu,\ \sigma^2$ from observations $x_1, \ldots, x_n$. with $\theta_1, \ldots, \theta_n,\ \sigma_x^2$ unknown. (you may assume independent priors, do not specify them, do not explain the distribution on the knots of the graph).

   b) sketch the collapsed graph for Bayesian inference on $\mu,\ \sigma^2$ if $\theta_1, \ldots, \theta_n$ are observed.

   c) describe the Gibbs sampler for drawing random numbers from the posterior distribution of (μ, σ^2) for problem **b)** (you do not need to compute explicitly the distribution on the knots of the graph).

d) continuation of **c)**: expert opinion: "μ around 50, certainly positive, but not as large as 100". What prior on μ, σ^2 do you propose (see **e)**)?

e) compute the full conditionals required for Gibbs sampling explicitly, using the prior in **d)**

3. time changing voting patterns: define p_t : population % of "yes" voters for a certain candidate at time t.

   model: $p_t = p^{(1)}$ for $t = t_1, \ldots, t_\kappa$, $p_t = p^{(2)}$ for $t = t_{\kappa+1}, \ldots, t_n$

   data: draw independently a sample voters each time and ask their preferences

time	t_1	$\cdots$	t_n
"yes" counts	x_1	$\cdots$	x_n
sample size	m	$\cdots$	m

   problem: Bayesian inference on $\left(p^{(1)}, p^{(2)}, \kappa\right)$

   a) find the likelihood function

   b) define priors on the unknown parameters of interest

   c) find posterior distribution

   compute posterior by Gibbs sampling:

   d) identify full conditionals

   e) write down the Gibbs sampler (algorithm) explicitly

4. Given a sample $(x_1, y_1), \ldots, (x_{15}, y_{15})$ iid $\sim F$, where F is a distribution with positive support: $F(x, y) > 0 \iff x > 0$ and $y > 0$. Define the parameter of interest $\eta = \mathbb{E}\frac{Y}{X}$.

   Using the bootstrap method:

   a) write an algorithm to construct a 90% percentile bootstrapped conf int for η

   b) give plausible reasons why this method described in **a)** should work

   c) write an algorithm to construct a 90% bootstrap-t-interval

   d) describe a situation when interval **c)** could be problematic while **a)** remains valid.